A Picture Is Worth a Thousand Words
Understanding Quantities and Their Relationships

**Vocabulary**
Write a definition for each term in your own words.

1. **independent quantity**
   The quantity that the dependent quantity depends on is the independent quantity.

2. **dependent quantity**
   When one quantity depends on another in a problem situation, it is said to be the dependent quantity.

**Problem Set**
Determine the independent and dependent quantities in each scenario.

1. Selena is driving to visit her grandmother who lives 325 miles away from Selena’s home. She travels an average of 60 miles per hour.
   - Independent quantity: time (hours)
   - Dependent quantity: distance (miles)

2. Benjamin works at a printing company. He is making T-shirts for a high school volleyball team. The press he runs can imprint 3 T-shirts per minute with the school’s mascot.
   - Independent quantity: time (minutes)
   - Dependent quantity: number of T-shirts imprinted

3. On her way to work each morning, Sophia purchases a small cup of coffee for $4.25 from the coffee shop.
   - Independent quantity: number of cups
   - Dependent quantity: cost (dollars)
4. Phillip enjoys rock climbing on the weekends. At some of the less challenging locations he can climb upwards of 12 feet per minute.
   Independent quantity: time (minutes)
   Dependent quantity: distance climbed (feet)

5. Jose prefers to walk to work when the weather is nice. He walks the 1.5 miles to work at a speed of about 3 miles per hour.
   Independent quantity: time (hours)
   Dependent quantity: distance (miles)

   Independent quantity: number of jumps
   Dependent quantity: cost (dollars)

Choose the graph that best models each scenario.

7. Kylie is filling her backyard pool to get ready for the summer. She is using a garden hose to fill the pool at a rate of 14 gallons per minute.

   [Graph A, Graph B, Graph C]

   Graph A

8. Hector is training to participate in competitive trampoline. In his best jump, he can reach a maximum height of about 9 meters and can spend about 2 seconds in the air performing tricks.

   [Graph A, Graph B, Graph C]

   Graph C
9. Jasmine is saving for college. She has invested $500 in a mutual fund that is expected to earn an average of 7% annually.

10. Each day Maria starts her walk to school at 7:45 AM. At 7:50 AM she stops at her friend Jenna's house. Jenna is usually late and Maria must wait at least 5 minutes for her to get ready. At 7:55 AM Maria and Jenna leave Jenna's house and arrive at school at 8:10 AM.
11. Marcus is at the top of an observation tower. He drops an action figure with a parachute attached and watches it descend to the ground.

Graph A

Graph B

Graph C

12. Janelle holds a raffle to raise money for a children’s hospital. Participants who enter the raffle guess the number of peanuts in a jar. Janelle records the number of peanuts each participant guesses and the number of peanuts their guess is off by.

Graph A

Graph B

Graph C
Label the axes of the graph that models each scenario with the independent and dependent quantities.

13. Madison enjoys bicycling for exercise. Each Saturday she bikes a course she has mapped out around her town. She averages a speed of 12 miles per hour on her journey.

14. Natasha is filling the bathtub with water in order to give her dog Buster a bath. The faucet fills the tub at an average rate of 12 gallons per minute.
15. Marcus throws a football straight up into the air. After it reaches its maximum height of 20 feet, it descends back to the ground.

16. Chloe is using a pump to drain her backyard pool to get ready for winter. The pump removes the water at an average rate of 15 gallons per minute.
17. Jermaine is saving money to purchase a used car. He places $850 dollars in a savings account that earns 1.65% interest annually.

![Graph of Interest Earned over Time (years)](image)

18. Zachary enjoys hiking. On the first day of his latest hiking trip, he hikes through flat terrain for about 8 miles. On the second day, he hikes through very steep terrain for about 3 miles. On the third day he hikes through some hilly terrain for about 6 miles.

![Graph of Distance Hiked over Time (days)](image)
## Vocabulary

Match each definition to its corresponding term.

1. A graph with no breaks in it  
   - b. continuous graph
2. The mapping between a set of inputs and a set of outputs  
   - c. relation
3. The set of all input values of a relation  
   - e. domain
4. The set of all output values of a relation  
   - f. range
5. A graph of isolated points  
   - a. discrete graph
6. A visual method used to determine whether a relation represented as a graph is a function  
   - g. Vertical Line Test
7. A relation between a given set of elements for which each input value there exists exactly one output value  
   - d. function
Problem Set

Each pair of graphs has been grouped together. Provide a rationale to explain why these graphs may have been grouped together.

1. Graph A

![Graph A](image1)

Graph B

![Graph B](image2)

Answers will vary.
Both graphs are always decreasing from left to right. Both graphs are functions. Both graphs are made up of straight lines.

2. Graph A

![Graph A](image3)

Graph B

![Graph B](image4)

Answers will vary.
Both graphs are smooth curves. Both graphs are functions. Both graphs have an increasing and a decreasing interval. Both graphs are U-shaped.
3. Graph A  Graph B

Answers will vary.
Both graphs have an increasing and a decreasing interval. Both graphs have a minimum value.
Both graphs are functions.

4. Graph A  Graph B

Answers will vary.
Both graphs have discrete data values. Both graphs are decreasing from left to right. Both graphs are functions.
5. Graph A

Answers will vary.
Both graphs are increasing from left to right. Both graphs are functions.

6. Graph A

Answers will vary.
Both graphs have an increasing and a decreasing interval. Both graphs have a maximum value. Both graphs are functions.
Determine whether the graph is discrete or continuous.

7. The graph is discrete.

8. The graph is continuous.

9. The graph is continuous.

10. The graph is continuous.

11. The graph is discrete.

12. The graph is continuous.
Determine if each graph represents a function by using the Vertical Line Test.

13. Yes. The graph is a function.

14. No. The graph is not a function.

15. No. The graph is not a function.

16. Yes. The graph is a function.
17. No. The graph is not a function.

18. Yes. The graph is a function.
There Are Many Ways to Represent Functions
Recognizing Algebraic and Graphical Representations of Functions

Vocabulary
Choose the term from the box that best completes each statement.

<table>
<thead>
<tr>
<th>term</th>
<th>function notation</th>
<th>increasing function</th>
<th>exponential functions</th>
<th>function family</th>
<th>linear functions</th>
<th>linear absolute value functions</th>
<th>absolute maximum</th>
<th>quadratic functions</th>
<th>constant function</th>
<th>linear piecewise functions</th>
<th>decreasing function</th>
<th>absolute minimum</th>
</tr>
</thead>
</table>

1. Function notation is a way to represent equations algebraically that makes it more efficient to recognize the independent and dependent variables.

2. The family of exponential functions includes functions of the form $f(x) = a \cdot b^x$, where $a$ and $b$ are real numbers, and $b$ is greater than 0 but is not equal to 1.

3. The family of linear piecewise functions includes functions that have an equation that changes for different parts, or pieces, of the domain.

4. When both the independent and dependent variables of a function increase across the entire domain, the function is called an increasing function.

5. A function has an absolute maximum if there is a point on its graph that has a $y$-coordinate that is greater than the $y$-coordinates of every other point on the graph.

6. A function family is a group of functions that share certain characteristics.

7. The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

8. When the dependent variable of a function decreases as the independent variable increases across the entire domain, the function is called a decreasing function.

9. The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

10. The family of linear functions includes functions of the form $f(x) = ax + b$, where $a$ and $b$ are real numbers, and $a$ is not equal to 0.

11. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a constant function.

12. A function has an absolute minimum if there is a point on its graph that has a $y$-coordinate that is less than the $y$-coordinates of every other point on the graph.
Problem Set
Rewrite each function using function notation.

1. Rewrite the function \( y = 3x - 8 \) using function notation so that the dependent quantity, defined as \( f \), is a function of the independent quantity \( x \).
   \[ f(x) = 3x - 8 \]

2. Rewrite the function \( y = 3x^2 + 6x - 1 \) using function notation so that the dependent quantity, defined as \( C \), is a function of the independent quantity \( x \).
   \[ C(x) = 3x^2 + 6x - 1 \]

3. Rewrite the function \( y = 3^x + 8 \) using function notation so that the dependent quantity, defined as \( P \), is a function of the independent quantity \( x \).
   \[ P(x) = 3^x + 8 \]

4. Rewrite the function \( l = |n - 2| \) using function notation so that the dependent quantity, defined as \( L \), is a function of the independent quantity \( n \).
   \[ L(n) = |n - 2| \]

5. Rewrite the function \( d = -\frac{1}{2}m + 5 \) using function notation so that the dependent quantity, defined as \( A \), is a function of the independent quantity \( m \).
   \[ A(m) = -\frac{1}{2}m + 5 \]

6. Rewrite the function \( c = 2\pi r^2 \) using function notation so that the dependent quantity, defined as \( C \), is a function of the independent quantity \( r \).
   \[ C(r) = 2\pi r^2 \]
Choose the graph that represents each function. Use your graphing calculator.

7. \( f(x) = \frac{2}{3}x + 2 \)

- **Graph A**
- **Graph B**
- **Graph C**

8. \( f(x) = -x^2 + 4 \)

- **Graph A**
- **Graph B**
- **Graph C**

9. \( f(x) = 2^x + 5 \)

- **Graph A**
- **Graph B**
- **Graph C**
10. \( f(x) = |x - 6| \)

Graph A

Graph B

Graph C

11. \( f(x) = 2x - 6 \), where \( x \) is an integer

Graph A

Graph B

Graph C

12. \( f(x) = -4 \)

Graph A

Graph B

Graph C
Determine whether each graph represents an increasing function, a decreasing function, a constant function, or a combination of increasing and decreasing functions.

13. The graph represents an increasing function.

14. The graph represents an increasing function.

15. The graph represents a function with a combination of an increasing interval and a decreasing interval.

16. The graph represents a decreasing function.

17. The graph represents a constant function.

18. The graph represents a decreasing function.
Determine whether each graph represents a function with an absolute minimum, an absolute maximum, or neither.

19. The graph represents a function with an absolute minimum.

20. The graph represents a function with neither an absolute minimum nor an absolute maximum.

21. The graph represents a function with an absolute maximum.

22. The graph represents a function with neither an absolute minimum nor an absolute maximum.

23. The graph represents a function with an absolute minimum.

24. The graph represents a function with an absolute minimum.
Name ____________________________________________ Date ____________

Determine whether each graph represents a linear function, a quadratic function, an exponential function, a linear absolute value function, a linear piecewise function, or a constant function.

25. The graph represents an exponential function.
26. The graph represents a linear function.
27. The graph represents a linear piecewise function.

28. The graph represents a quadratic function.
29. The graph represents a constant function.
30. The graph represents a linear absolute value function.
Function Families for 200, Alex…
Recognizing Functions by Characteristics

**Problem Set**

Choose the appropriate function family or families to complete each sentence based on the given characteristic(s).

<table>
<thead>
<tr>
<th>linear functions</th>
<th>quadratic functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponential functions</td>
<td>linear absolute value functions</td>
</tr>
</tbody>
</table>

1. The graph of this function family is a straight line. The function family is **linear functions**.

2. The graph of this function family has an increasing interval and a decreasing interval. The function family is **quadratic functions or linear absolute value functions**.

3. The graph of this function family has an absolute minimum. The function family is **quadratic functions or linear absolute value functions**.

4. The graph of this function family is decreasing over the entire domain. The function family is **linear functions or exponential functions**.

5. The graph of this function family forms a V shape. The function family is **linear absolute value functions**.

6. The graph of this function family has an increasing interval and a decreasing interval and forms a U shape. The function family is **quadratic functions**.

7. The graph of this function family does not have an absolute maximum or absolute minimum and is a smooth curve. The function family is **exponential functions**.

8. The graph of this function family has an absolute maximum or absolute minimum and is made up straight lines. The function family is **linear absolute value functions**.

9. The graph of this function family is made up straight lines and does not have an absolute maximum or absolute minimum. The function family is **linear functions**.

10. The graph of this function family decreases over the entire domain and is a smooth curve. The function family is **exponential functions**.
Create an equation and sketch a graph for a function with each set of given characteristics. Use values that are any real numbers between $-10$ and $10$.

11. Create an equation and sketch a graph that:
   - is a smooth curve,
   - is continuous,
   - has a minimum, and
   - is quadratic.

   Answers will vary.
   \[ f(x) = x^2 \]

12. Create an equation and sketch a graph that:
   - is linear,
   - is discrete, and
   - is decreasing across the entire domain.

   Answers will vary.
   \[ f(x) = -x, \text{ where } x \text{ is an integer} \]
13. Create an equation and sketch a graph that:
   - is a smooth curve,
   - is increasing across the entire domain,
   - is continuous, and
   - is exponential.

   Answers will vary.
   \[ f(x) = 2^x \]

14. Create an equation and sketch a graph that:
   - has a maximum,
   - is continuous, and
   - is a linear absolute value function.

   Answers will vary.
   \[ f(x) = |x| \]
15. Create an equation and sketch a graph that:
   • is linear,
   • is continuous,
   • is neither increasing nor decreasing across the entire domain, and
   • does not pass through the origin.

   Answers will vary.
   \[ f(x) = 3 \]

16. Create an equation and sketch a graph that:
   • is discrete,
   • has a maximum,
   • does not pass through the origin, and
   • is quadratic.

   Answers will vary.
   \[ f(x) = -x^2 + 3, \text{ where } x \text{ is an integer} \]
Choose the function family represented by each graph.

- linear function
- quadratic function
- exponential function
- linear absolute value function
- linear piecewise function

17. The graph represents a quadratic function.

18. The graph represents a linear function.

19. The graph represents a linear absolute value function.

20. The graph represents an exponential function.
21. The graph represents a linear piecewise function.

22. The graph represents a linear function.